COMMON PRE-BOARD EXAMINATION 2022-23

Class XII

Subject: APPLIED MATHEMATICS - 241

Date:

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

1) This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2) Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3) **Section – A:** It comprises of **20 MCQs of 1 mark** each.
4) **Section – B:** It comprises of **5 VSA type questions of 2 marks** each.
5) **Section – C:** It comprises of **6 SA type of questions of 3 marks** each.
6) **Section – D:** It comprises of **4 LA type of questions of 5 marks** each.
7) **Section – E:** It has **3 case studies**. Each case study comprises of 3 case-based questions, where **2 VSA type questions are of 1 mark** each and **1 SA type question is of 2 marks**. Internal choice is provided in 2 marks question in each case-study.
8) Internal choice is provided in **2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D**. You have to attempt only one of the alternatives in all such questions.

SECTION – A

(Multiple Choice Questions)

(Each question carries 1 mark)

1. What is the least value of x that satisfies $x \equiv 17 (\text{mod } 4)$, when $18 < x \leq 25$?
   a) 17 b) 21 c) 25 d) Not defined
2. Solution of the following inequality for real x is:
   $3(2-x) \geq 2(1-x)$
   a) $x \leq 6$ b) $x \leq 4$ c) $x \geq 4$ d) $x \geq 6$
3. A sample of 50 bulbs are taken at random. Out of 50 we found 15 bulbs from company A, 17 from B and 18 from C. What is the point estimate of population proportion of B?
   a) 0.3 b) 0.4 c) 0.36 d) 0.34
4. Degree of freedom if N is the sample size is
   a) $N-1$ b) $N+1$ c) $2N$ d) $\frac{N}{2}$
5. The speed of a boat in still water is 15 km/h and the rate of current in 3 km/h. The distance travelled by boat downstream in 12 minutes is
   a) 3.6 km b) 1.8 km c) 2.4 km d) 1.2 km
6. A specific characteristic of a population is known as a
   a) sample b) parameter c) statistic d) mean

7. A certain tank can be filled by pipe A in 12 minutes, pipe B can empty the tank in 18 minutes. If both pipes are open, then the time it takes to fill the tank is
   a) 5 minutes b) 6.4 minutes c) 7.2 minutes d) 8.5 minutes
8. Calculate present value of a sequence of payment ₹ 60 made at end of each 6 months and continuing forever, if money is worth 4% compounded semi - annually
   a) ₹ 3000 b) ₹ 1000 c) ₹ 300 d) ₹ 360
9. $\int(x^2 - e^x)dx =$
   a) $x^2 - e^x + C$ b) $2x^2 - e^x + C$ c) $\frac{x^3}{3} - e^x + C$ d) $\frac{x^3}{3} + e^x + C$
10. A machine costing 40,000 is expected to have a useful life of 4 years and a final scrap value of 8000. Find the annual depreciation charge using the straight-line method.
    a) 8000 b) 10000 c) 5000 d) 4000
11. A linear Programming Problem is as follows:
    Minimize: Z = 2x + y
    Subject to the constraints
    $x \geq 3, x \leq 9, y \geq 0, x - y \geq 0, x + y \leq 14$
    The feasible region has
    a) 5 corner points including (0,0) and (9,5) b) 5 corner points including (7,7) and (3,3)
    c) 5 corner points including (14,0) and (9,0) d) 5 corner points including (3,6) and (9,5)
12. Assume an investment's starting value is 2,00,000 and it grows to 12,00,000 in 4 years. Calculate CAGR
    a) 56.5% b) 50.5% c) 60.5% d) 60%
13. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} = 0$ is
    a) 1 b) 2 c) 3 d) 4
14. Mr. Anil takes a loan of Rs.2,00,000 with 10% annual interest rate for 5 years. EMI under flat rates system is
    a) 4000 b) 5000 c) 6000 d) 7000
15. In what ratio must rice at ₹ 69 per kg be mixed with rice ₹ 100 per kg so that the mixture be worth ₹ 80 per kg?
    a) 11:20 b) 20:11 c) 69:100 d) 100:69
16. Consider the following hypothesis test
    $H_0 : \mu \leq 25, H_1 : \mu > 25$
    A sample of 40 provided a sample mean of 26.4, then the value of the test statistics is:
    a) 4.18 b) - 1.48 c) 1.48 d) - 4.18
17. For the given five Values 15,24,18,33,42 the three years moving averages are
    a) 19,25,33 b) 19,30,31 c) 19,25,31 d) 19,22,33
18. Seasonal variation mean the variations occurred within
    a) A number of years b) Parts of a year c) Parts of a month d) None of these

ASSERTION-REASON BASED QUESTIONS

For questions 19 and 20, two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:

(i) **Both A and R are true and R is the correct explanation of the assertion**

(ii) **Both A and R are true but R is not the correct explanation of the assertion**

(iii) **A is true, but R is false**

(iv) **A is false, but R is true**

19. **Assertion (A):** If the nominal rate of interest is 12.5% and the inflation is 2%, then the effective rate of interest is 10.5%

Reason (R): If the interest is calculated only at the end of an year, then the effective rate of interest is same as the nominal rate of interest.

a) (i) b) (ii) c) (iii) d) (iv)

20. **Assertion (A):** Given that mean of a normal variate X is 12 and standard deviation is 4, then the Z-Score of data point 20 is 12.

Reason (R): When mean = 0 and standard deviation = 1 for a data set, then the normal distribution is called as standard normal distribution.

a) (i) b) (ii) c) (iii) d) (iv)

SECTION – B

(All questions are compulsory. In case of internal choice, attempt any one question only)

21. How much money is needed to endure a series of lectures costing ₹ 2500 at the beginning of each year indefinitely, if money is worth 3% compounded annually?

22. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then find the value of k.

OR

Find the inverse of matrix $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

23. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes on hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹ 300. Formulate a L.P.P for finding how many of each should be produced daily to maximize the profit?

24. In a 500 m race, A defeats B by 60 meters (or) 12 seconds. What is the time taken by A to complete the race?

OR

A man can swim a certain distance downstream in 3 hours and returns the same distance upstream in 6 hours. If the speed of stream is 2 km/h, then find the speed of man in still water.

25. Mr. Ajay took a loan of 2,000 for 6 months. Lender deducts 200 as interest while lending. Find the effective rate of interest charged by lender.

SECTION – C

(All questions are compulsory. In case of internal choice, attempt any one question only)

26. Find the intervals in which the function $f(x) = 2x^3 + 9x^2 + 12x + 20$ is

(i) Increasing (ii) decreasing

27. Express the following matrices as the sum of a symmetric and a skew symmetric matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

28. Evaluate: $\int \frac{(2x+1)}{(x+1)(x-2)} dx$

OR

Evaluate: $\int \frac{x^2}{(x-1)(x-2)(x-3)} dx$

29. If a manufacturer's total cost function C is given by $C = \frac{x^2}{25} + 2x$, find

(i) average cost function

(ii) the marginal cost function

(iii) the marginal cost when 5 units are produced.

OR

The supply function for a commodity is $p = x^2 + 4x + 5$ where x denotes supply. Find the producers' surplus when the price is 10.

30. Mr. Ram has set up a sinking fund so that he can accumulate ₹ 10,00,000 in 10 years for his Children's higher education. How much should he deposit every six months if interest is 5% per annum compounded semi-annually? (Use $(1.025)^{20} = 1.6386$)

31. Surjeet purchased a new house, costing ₹ 40,00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly instalment is ₹ 30,000, at 9% interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him? [Use $(1.0075)^{-300} = 0.1062$].

SECTION – D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. If 5% of the electric bulbs manufactured by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs: ($e^{-5} = 0.007$)

(i) None is defective

(ii) 5 bulbs will be defective

(iii) At least one is defective

OR

In an examination, 2000 students appeared and the mean of the normal distribution of marks is 30 with standard deviation as 6.25. Find out how many students are expected to score

(i) between 20 and 40 marks

(ii) less than 25 marks

33. The following data shows the percentage of urban Indian who have a high-speed internet connection at home

Year	2015	2016	2017	2018	2019
Urban	9	18	21	29	38

(i) Based on the above information use the method of least-squares to find the best-fit trend line equation for Urban Indians.

(ii) compute expected percentage for year 2020

OR

Compute the trends by the method of moving averages, assuming that 4-year cycle is present in the following series.

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Index Number	400	470	450	410	432	475	461	500	480	430

34. Maximize: $Z = 10500x + 9000y$

Subject to constraints: $x + y \leq 50, 2x + y \leq 80, x, y \geq 0$

35. Solve the system of equations using matrix method
$2x - 3y + 5z = 11$, $3x + 2y - 4z = 5$, $x + y - 2z = -3$

SECTION – E

(All questions are compulsory. In case of internal choice, attempt any one question only)

36. **Case Study – I:**

In a survey of studies, Let X be the random variable which count the number of hours a student of class XII studied. X has a probability distribution P (X) of the following form

X	0	1	2	3	4
P (X)	k	2k	3k	4k	5k

Based on the above information, answer the following questions.

a) Find the value of k.
b) What is the probability that student studies 1 hour.
c) Calculate the mathematical expectation of number of hours studied by student.

OR

What is the probability that a student study at least 3 hours?

37. **Case Study – II:**

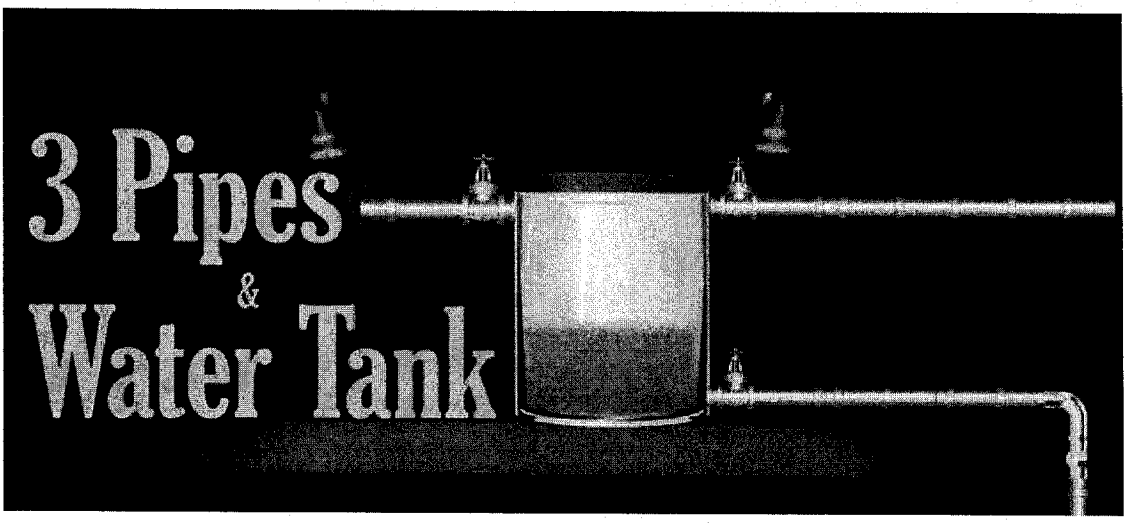
To manufacture 'x' number of dolls, a company's total cost function C(x) is given by C (x) = $100 + 0.025x^2$ and the total revenue function R (x) is described as R (x) = $5x$. C (x) and R (x) are in thousand rupees.

a) Find profit function.
b) Find critical point.
c) What is the maximum profit?

OR

Find the marginal cost when 250 dolls are produced.

38. A pipe is connected to a tank. It is used to fill or empty the tank. The amount of work done by a pipe is a part of the tank filled or emptied in unit time.
Three pipes A, B and C are connected to a tank. A and B fill the tank in 6 and 8 hours respectively when operated independently. Pipe C empties the full tank in 12 hours when opened alone.



Based on the above information, answer the following questions:

a) If pipe A and C are opened together, then find the time taken to fill the tank.
b) If pipe B and C are opened together, then find the time taken to fill the tank.
c) If both pipes A and B are opened together, then find the time taken to fill the tank

OR

If three pipes are opened together, then find the time taken to fill the tank.